# VERTICAL FORCES ACTING ON HORIZONTAL DISKS 

## IN A FLUIDIZED BED

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Experimental studies of vertical forces were made with a turbulimeter. A dimensionless expression was obtained for the determination of the average vertical forcing acting on horizontal disks submerged in a fluidized bed.

In setting up fragile heat-treated components (ceramic forms intended for precision casting from fusible patterns, thin-walled "sovelite" sheets, thin aluminum sheets) in apparatuses having a fluidized bed, it is necessary to know the forces acting on the components which can lead to their rupture or buckling in order to design components having the proper strength. There is little data on the forces in the literature and no generalized results. The aim of the present work is the study and generalization of data for the simple "canonical" case of horizontal disks having an area not exceeding $10 \%$ of the area of the apparatus.

The vertical force $K_{Z}$ acting on a body submerged in a fluidized bed can depend on the following independent variables [1, 2]:

$$
\begin{equation*}
K_{z}=f\left(w, D_{\mathrm{b}} v_{\mathrm{fa}} \rho_{\mathrm{fa}} \rho_{\mathrm{fm}} g, d_{\mathrm{e}}, H_{\mathrm{t}}, H_{0}, t, l\right) \tag{1}
\end{equation*}
$$

Investigations of the pulsational characteristics of heat transfer in a fluidized bed [3] show that the viscosity $\nu_{\mathrm{fa}}$ and the density $\rho_{\mathrm{fa}}$ of the fluidizing agent affect the hydrodynamic situation at the surface of the body around which flow occurs only in so far as they affect the critical velocity of fluidization wf. Phyically this is understandable, because from the position of the two-phase theory of nonuniform fluidization [4], the hydrodynamics of the bed depend on the volume, number, and velocity of the bubbles, which are determined by the excess velocity ( $w-w f$ ) of the fluidizing agent, the particle diameter $d_{e}$, and the height $\mathrm{H}_{\mathrm{t}}$ above the grid. In analogy with [3], therefore, it is convenient to select the Froude number $\mathrm{Fr}=$ ( w $\left.-\mathrm{w}_{\mathrm{f}}\right)^{2} / \mathrm{gd}_{\mathrm{e}}$ as one of the defining criteria without considering the effects of $\nu_{\mathrm{f} a}, \rho_{\mathrm{fa}}$, and $w$ separately. With an increase in the spacing $t$ between the caps of the gas-distribution grid, large-scale circulation of material because of deterioration of the equilibrium of gas distribution over the cross section of the apparatus leads to a change in $\mathrm{K}_{\mathrm{Z}}$. The effect of the height $\mathrm{H}_{\mathrm{t}}$ of the turbulimeter above the gas distribution grid and the thickness $\mathrm{H}_{0}$ of the bed is explained by the increase in bubble size and velocity with height [5]. The acceleration of gravity $g$ has an effect only in so far as the rate of bubble rise depends on its value. The effects of $d_{e}, D_{b}$, and $\rho_{\mathrm{fm}}$ are obvious.

Besides the factors mentioned, the size and shape of the cross section of the device may have an effect, as well as the location of the body in the cross section of the apparatus. The resistive force depends on the shape of the body and also on its orientation in the bed. In this paper, we confine ourselves to consideration forces acting on thin $(l=0)$ horizontal disks located at the center of the cross section of the apparatus.

The studies were made in an apparatus with a rectangular cross section $0.3 \times 0.15 \mathrm{~m}$ in size and 1 m high. The gas-distribution system consisted of 32 caps with a $1 \%$ fractional useful cross section. The turbulimeter [6, 7] used for measurement of dynamic forces consisted of a steel (U8 steel) uniform-resistance arm rigidly fastened at its base having strain gauges with a $10-\mathrm{mm}$ base and $180-\Omega$ resistance glued to it. Test disks were rigidly fastened to the free end of the arm. The strain gauges were connected in the bridge circuit of a strain test stand, the output of which was sent to the input of an $\mathrm{MN}-7 \mathrm{M}$ analog computer. The signal was sent in parallel to the galvanometer of an $\mathrm{N}-700$ loop oscillograph, where it was
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Fig. 1


Fig. 2

Fig. 1. Dependence of average force $K_{Z}, N$, on Froude number for various corundum fractions: $\mathrm{H}_{\mathrm{t}}=0.35 \mathrm{~m} ; \mathrm{H}_{0}=0.5 \mathrm{~m} ; \mathrm{D}_{\mathrm{b}}=0.03 \mathrm{~m}$; 1) $\mathrm{d}_{\mathrm{e}}=74 \mu ; 2$ ) $196 \mu$; 3) $212 \mu$; 4) $245 \mu$; 5) $290 \mu$; 6) $400 \mu$; 7) $695 \mu$.
Fig. 2. Comparison of some experimental data with the theoretical relation (4): 1) $\mathrm{d}_{\mathrm{e}}=400 \mu, \mathrm{H}_{\mathrm{t}}=0.35 \mathrm{~m}, \mathrm{H}_{0}=0.5 \mathrm{~m}, \mathrm{t}=0.033 \mathrm{~m}, \mathrm{D}_{\mathrm{b}}=0.03 \mathrm{~m}$; 2) corresponding values are $400,0.35,0.5,0.033,0.08 ; 3$ ) corresponding values are $196,0.5,0.5,0.033,0.03 ; 4$ ) corresponding values are $196,0.15$, $0.15,0.033,0.03 ; 5$ ) corresponding values are $196,0.15,0.15,0.033,0.06$; 6) corresponding values are $400,0.5,0.5,0.033,0.02 ; 7$ ) corresponding values are $212,0.35,0.5,0.033,0.03 ; 8$ ) corresponding values are $290,0.35$, $0.5,0.099,0.03 ; 9$ ) corresponding values are $760,0.35,0.5,0.033,0.03$; 1-8) corundum; 9) chamotte.
recorded on photographic paper for 10 sec . For the selected integration time of the $\mathrm{MN}-7 \mathrm{M}$ ( 2 min ), the mathematical expectation with respect to various realizations differed by no more than $3 \%$, which also determined the accuracy of the vertical force measurements. We investigated the forces which acted on horizontal disks with diameters $\mathrm{D}_{\mathrm{b}}=0.02,0.03,0.04,0.06$, and 0.08 m located in the bed at heights $\mathrm{H}_{\mathrm{f}}$ $=0.15,0.35$, and 0.50 m from the exit openings in the caps for bed thicknesses $\mathrm{H}_{0}=0.15,0.35$, and 0.50 m (the useful cross section of the openings remained unchanged and was equal to $1 \%$; when the spacing $t$ between caps was increased from 0.033 to 0.099 m , the number of openings in the caps was increased). We used corundum particles with equivalent diameters $d_{e}=74,196,212,245,290,400$, and $696 \mu$ and a density $\rho_{\mathrm{fm}}=3880 \mathrm{~kg} / \mathrm{m}^{3}$, chamotte particles with $\mathrm{d}_{\mathrm{e}}=760 \mu$ and $\rho_{\mathrm{fm}}=1810 \mathrm{~kg} / \mathrm{m}^{3}$, and magnesium oxide particles with $\mathrm{d}_{\mathrm{e}}=500 \mu$ and $\rho_{\mathrm{fm}}=3270 \mathrm{~kg} / \mathrm{m}^{3}$.

The function $K_{Z}=f(F r)$ which was obtained for a large range of fluidizing-agent velocities with the diameter of corundum particles varying from 74 to $695 \mu$, is expressed by the following power function:

$$
\begin{equation*}
K_{z} \sim\left[\frac{\left(w-w_{f}\right)^{2}}{g d_{\mathrm{e}}}\right]^{0, \overline{5}} . \tag{2}
\end{equation*}
$$

The equivalent particle diameter $d_{e}$ is calculated from

$$
\begin{equation*}
\frac{1}{d_{\mathrm{e}}}=\sum \frac{X_{i}}{d_{i}}, \tag{3}
\end{equation*}
$$

where $X_{i}$ is the mass fraction of particles in a narrow band around the diameter $d_{i}$. The minimum fluid--ization velocity $w_{f}$ was determined from the Todes formula [8] for a porosity $\varepsilon_{0}=0.4$. Since all experimental points are well described by Eq. (2) (Fig. 1) independently of particle diameter, one need not additionally take into account the effect of $\mathrm{d}_{\mathrm{e}}$ on $\mathrm{K}_{\mathrm{Z}}$, other conditions being equal.

Having constructed the corresponding dependences of $K_{Z}$ on Froude number for varying $\mathrm{Db}_{\mathrm{b}}, \mathrm{H}_{\mathrm{t}}, \mathrm{H}_{0}$, and $t$, and having determined the numerical values of the slopes of the corresponding straight lines passing through the experimental points, we have succeeded in obtaining the following dimensionless formula for the determination of the average vertical force on a body submerged in a fluidized bed:

$$
\begin{equation*}
\frac{K_{z}}{g D_{\mathrm{b}}^{3} \rho_{\mathrm{fm}}}=12.8\left[\frac{\left(w-w_{\mathrm{f}}\right)^{2}}{g d_{\mathrm{e}}}\right]^{0.5}\left(\frac{H_{\mathrm{t}}}{D_{\mathrm{b}}}\right)^{0.86}\left(\frac{H_{0}}{D_{\mathrm{b}}}\right)^{0.38}\left(\frac{t}{D_{\mathrm{b}}}\right)^{0.28} \tag{4}
\end{equation*}
$$

Some experimental data shown in Fig. 2 for comparison are in good agreement with the theoretical relation (4). Here

$$
\begin{gather*}
Y=\frac{K_{x}}{g D_{\mathrm{b}}^{3} \rho_{\mathrm{fm}}}  \tag{5}\\
X=\operatorname{Fr}^{0.5}\left(\frac{H_{\mathrm{t}}}{D_{\mathrm{b}}}\right)^{0.86}\left(\frac{H_{0}}{D_{\mathrm{b}}}\right)^{0.38}\left(\frac{t}{D_{\mathrm{b}}}\right)^{0.28} \tag{6}
\end{gather*}
$$

The approximation obtained is valid for the following limits on parameter ranges: $\mathrm{Fr}=1-250 ; \mathrm{H}_{\mathrm{t}} / \mathrm{D}_{\mathrm{b}}=2-$ $25 ; \mathrm{t} / \mathrm{D}_{\mathrm{b}}=0.4-5 ; \mathrm{H}_{0} / \mathrm{D}_{\mathrm{b}}=2-25$. The range of $\mathrm{K}_{\mathrm{z}}$ is $6.5 \mathrm{~m} \cdot \mathrm{~kg} / \mathrm{sec}^{2}$.

It should be particularly emphasized that Eq. (4) is only valid for the calculation of the time-averaged vertical force acting on a body located in the center of the cross section of the apparatus. When the body is shifted closer to the walls of the apparatus, the force $\mathrm{K}_{\mathrm{Z}}$ is correspondingly reduced and is directed downwards where there is a descending motion of the material. This is typical of apparatuses with a fluidized bed in which the solid particles are displaced upwards by the rising bubbles predominantly in the center of the apparatus and descending motion of the material occurs at the walls.

In strength design for bodies submerged in a bed, it is necessary to keep in mind the maximum short-time loading, which is greater than the average force. The ratio between the maximum load acting on a submerged body in the upward direction and the average force decreases from $15-11$ to 8-7, respectively, for horizontal disks $0.02-0.04 \mathrm{~m}$ in diameter as the fluidization velocity increases. For horizontal disks 0.06 and 0.08 m in diameter, this ratio becomes practically constant and is 5.5-6.5. The ratio between the maximum load acting on a submerged body in the downwards direction and the average force changes from 6.5-3 to $3-2$, respectively, for horizontal disks $0.08-0.02 \mathrm{~m}$ in diameter as the fluidization velocity increases.

## NOTATION

| $\mathrm{K}_{\mathrm{z}}$ | is the mean vertical force, $\mathrm{m} \cdot \mathrm{kg} / \mathrm{sec}^{2} ;$ |
| :--- | :--- |
| w | is the fluidization velocity at free section of the apparatus, $\mathrm{m} / \mathrm{sec} ;$ |
| $\mathrm{D}_{\mathrm{b}}$ | is the diameter of body, $\mathrm{m} ;$ |
| $\nu_{\mathrm{fa}}$ | is the kinematic viscosity of fluidizing agent, $\mathrm{m}^{2} / \mathrm{sec} ;$ |
| $\rho_{\mathrm{fa}}, \rho_{\mathrm{fm}}$ | are the density of fluidizing agent and fluidized material, $\mathrm{kg} / \mathrm{m}^{3} ;$ <br> g |
| is the acceleration of gravity, $\mathrm{m} / \mathrm{sec} ;$ |  |
| $\mathrm{d}_{\mathrm{e}}$ | is the equivalent particle diameter, $\mathrm{m} ;$ |
| $\mathrm{H}_{\mathrm{t}}$ | is the height of turbulimeter position in the bed calculated from exit holes in caps, $\mathrm{m} ;$ |
| $\mathrm{H}_{0}$ | is the height of packing, $\mathrm{m} ;$ |
| $l$ | is the thickness of body, $\mathrm{m} ;$ |
| t | is the space between caps, m. |

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